Edexcel Maths FP2

Topic Questions from Papers

2nd Order Differential Equations

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8.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

(a) find x in terms of t.

(8)

The solution to part (a) is used to represent the motion of a particle P on the x-axis. At time t seconds, where $t > 0$, P is x metres from the origin O.

(b) Show that the maximum distance between O and P is $\frac{2\sqrt{3}}{9}$ m and justify that this distance is a maximum.

(7)

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Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

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8. (a) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x$$

(4)

(b) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x$$

(3)

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$.

(5)

(d) Sketch the curve with equation $y = f(x)$ for $0 \leqslant x \leqslant \pi$.

(2)

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

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8. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \quad t \geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Find the general solution of this differential equation.

(8)

(b) Find the particular solution of this differential equation for which, at $t = 0$,

$x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$.

(5)

On the graph of the particular solution defined in part (b), the first turning point for $t > 30$ is the point A.

(c) Find approximate values for the coordinates of A.

(2)

Leave blank

Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

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4. Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2\cos t - \sin t$$

(9)

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Question 4 continued

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7. (a) Find the value of λ for which $\lambda t^2 e^{3t}$ is a particular integral of the differential equation

$$\frac{d^2 y}{dt^2} - 6\frac{dy}{dt} + 9y = 6e^{3t}, \qquad t \geqslant 0$$

(5)

(b) Hence find the general solution of this differential equation.

(3)

Given that when $t = 0$, $y = 5$ and $\dfrac{dy}{dt} = 4$

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(t)$.

(5)

Leave blank

Question 7 continued

P 4 2 9 5 5 A 0 2 5 3 2

Turn over

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7. (a) Show that the transformation $y = xv$ transforms the equation

$$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + (8 + 4x^2)y = x^4 \qquad \text{(I)}$$

into the equation

$$4\frac{d^2v}{dx^2} + 4v = x \qquad \text{(II)}$$

(6)

(b) Solve the differential equation (II) to find v as a function of x.

(6)

(c) Hence state the general solution of the differential equation (I).

(1)

Leave blank

Question 7 continued

Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2}\int r^2 \, \mathrm{d}\theta \qquad \text{(polar coordinates)}$$

Complex numbers

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k \mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's and Taylor's Series

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{f}(x) = \mathrm{f}(a) + (x-a)\,\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \ldots + \frac{(x-a)^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{f}(a+x) = \mathrm{f}(a) + x\,\mathrm{f}'(a) + \frac{x^2}{2!}\mathrm{f}''(a) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leq x \leq 1)$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\lvert\tan(\frac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\lvert\tan(\frac{1}{2}x + \frac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\text{cosec}^2\, x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$